# 6.2 Energy of Orbiting Satellites

**SECTION EXPECTATIONS**

- Analyze the factors affecting the motion of isolated celestial objects, and calculate the gravitational potential energy for such a system.
- Analyze isolated planetary and satellite motion and describe it in terms of the forms of energy and energy transformations that occur.
- Calculate the kinetic and gravitational potential energy of a satellite that is in a stable circular orbit around a planet.

**KEY TERMS**

- circular orbit
- total orbital energy

The rockets that launched *Voyager 1* and *Voyager 2,* were designed to escape Earth's gravity and send them into space to search the solar system. The *Voyager* craft have found such things as new moons orbiting Jupiter, Saturn, Uranus, and Neptune. They have also discovered volcanoes on Io and rings around Jupiter. However, the majority of satellites are launched into Earth's orbit and will remain captive in Earth's gravitational field, destined to circle the planet year after year and perform tasks of immediate importance to people on Earth.

## Satellites in Earth Orbit

Some of these satellites monitor the weather, the growth and health of crops, the temperature of the oceans, the presence of ice floes, and the status of the ozone layer. Others actively scan Earth's surface with radar to enhance our knowledge of the geography and geology of our planet. These days, many people routinely use satellites to tell them where they are (for example, the Global Positioning System) and to provide them with mobile communication and seemingly limitless television entertainment.

Other satellites look outward, monitoring regions of the electromagnetic spectrum that cannot pass easily through Earth's atmosphere. In doing so, they tell us about our own solar system, as well as other solar systems, stars, and galaxies located many light-years away from us.

The largest artificial satellite in Earth's orbit is the International Space Station. During its construction and lifetime, it has been serviced from other temporary satellites, the space shuttles. For these shuttles to rendezvous successfully with the space station, teams of scientists, engineers, and technicians must solve problems involving the orbital motions and energies that are the subject of this section.

Most satellites are in either a **circular orbit** or a near-circular orbit. In Chapter 3, you learned how the force of gravity acts as a centripetal force, holding each satellite in its own unique orbit. You learned how to calculate the orbital speeds of the satellites that orbit at specific radii. In this section, you will focus on the energies of these satellites.

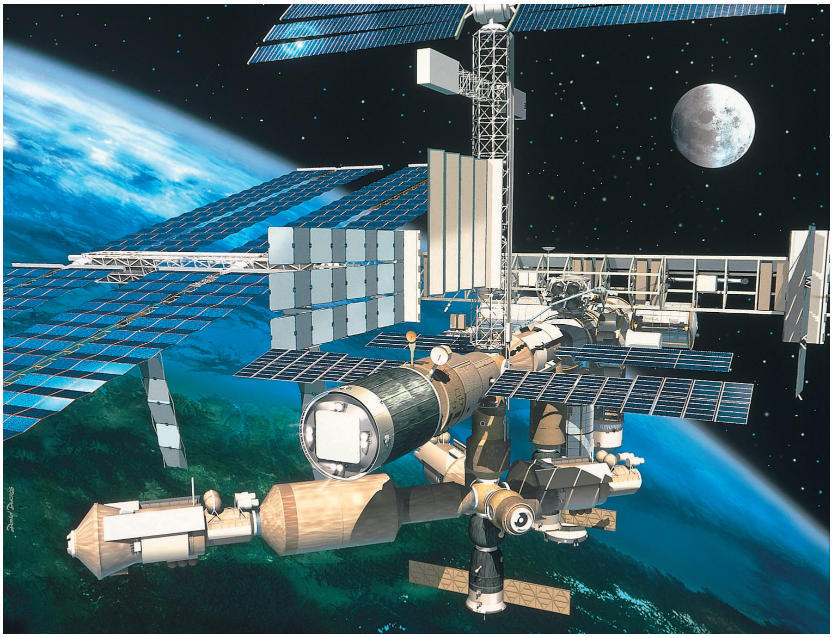


**Figure 6.2** A thorough understanding of orbital mechanics is necessary for a successful rendezvous with the space station or with any other satellite.

## Orbital Energies

The orbital energy of a satellite consists of two components: its kinetic energy and its gravitational potential energy. To a great extent, the Earth-satellite system can be treated as an isolated system. Subtle effects, such as the pressure of light, the solar wind, and collisions with the few atmospheric molecules that exist at that distance from the surface, can change the energy of the system. In fact, without the occasional boost from a thruster, the orbits of all satellites will decay. However, this generally takes decades. These effects are so tiny over the short run that you will neglect them in the following topics.

- Write the relationship that represents a planet's gravity providing a centripetal force.

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

- Multiply both sides of the equation by $r$.

$$\frac{mv^2\cancel{r}}{\cancel{r}} = \frac{GMm}{r^{\cancel{2}}}\cancel{r}$$

$$mv^2 = \frac{GMm}{r}$$

- Multiply both sides of the equation by $\frac{1}{2}$.

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

- Since $\frac{1}{2}mv^2$ is the kinetic energy of any object of mass $m$, you can substitute $E_k$ for the expression.

$$E_k = \frac{GMm}{2r}$$

The kinetic energy of an orbiting satellite of mass $m$ is $E_k = \dfrac{GMm}{2r}$.

## Gravitational Potential Energy

In Chapter 5, Conservation of Energy, you demonstrated that the change in the gravitational potential energy of an object was equal to the work done in raising the object from one height to another. That relationship ($W = mg\Delta h$) was the special case, where any change in height was very close to Earth's surface. Since you are now dealing with objects being launched into space, you cannot use the special case. You must consider the change in the force of gravity as the distance from Earth increases. Fortunately, however, you have already developed an expression for the amount of work required to lift an object from a distance $r_1$ to a distance $r_2$ from Earth's centre. Therefore, the result of your derivation is equal to the change in the gravitational potential energy between those two positions.

$$\Delta E_g = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

As you know, you must choose a reference point for all forms of potential energy. Earth's surface in no longer an appropriate reference, because you are measuring distances from Earth's centre to deep into space. Physicists have accepted the convention of assigning the reference or zero point for gravitational potential energy as an infinite distance from the centre of the planet or other celestial body that is exerting the gravitational force on the object of mass $m$. This is appropriate because at an infinite distance, the gravitational force goes to zero. You can now state that the gravitational potential energy of an object at a distance $r_2$ from Earth's centre is the amount of work required to move an object from an infinite distance, $r_1$, to $r_2$.

$$E_g = GMm\left(\frac{1}{\infty} - \frac{1}{r_2}\right)$$

$$E_g = GMm\left(0 - \frac{1}{r_2}\right)$$

$$E_g = -\frac{GMm}{r_2}$$

Since there is only one distance ($r_2$) in the equation, it is often written without a subscript. Notice, also, that the value is negative. This is simply a result of the arbitrary choice of an infinite distance for the reference position. You will discover as you work with the concept that it is a fortunate choice.

### GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy of an object is the negative of the product of the universal gravitational constant, the mass of the planet or celestial body, and the mass of the object, divided by the distance from the centre of the planet or celestial body.

$$E_g = -\frac{GMm}{r}$$

| Quantity | Symbol | SI unit |
|---|---|---|
| gravitational potential energy | $E_g$ | J (joules) |
| universal gravitational constant | $G$ | $\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{kg}^2}$ (newton metres squared per kilograms squared) |
| mass of the planet or celestial body | $M$ | kg (kilograms) |
| mass of the object | $m$ | kg (kilograms) |
| distance from centre of planet or celestial body | $r$ | m (metres) |

**Unit Analysis**

$$\text{joule} = \frac{\frac{\text{newton}\cdot\text{metre}^2}{\text{kilogram}^2}\cdot\text{kilogram}\cdot\text{kilogram}}{\text{metre}}$$

$$\mathrm{J} = \frac{\frac{\mathrm{N}\cdot\mathrm{m}^{\cancel{2}}}{\cancel{\mathrm{kg}^2}}\cdot\cancel{\mathrm{kg}}\cdot\cancel{\mathrm{kg}}}{\cancel{\mathrm{m}}} = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

**Note:** Use of this equation implies that the reference or zero position is an infinite distance from the planet or celestial body.

It might seem odd that the potential energy is always negative. Since changes in energy are always of interest, however, these changes will be the same, regardless of the location of the zero level.

To illustrate this concept, consider the houses in the Loire Valley in France that are carved out of the face of limestone cliffs as shown in Figure 4.3(B). To the person on the cobblestone street, everyone on floors A, B, and C in Figure 6.3(A) would have positive gravitational energy, due to their height above the street. However, to a person on floor B, those on floor A are at a negative height, and so have negative gravitational potential energy relative to them. At the same time, the person on floor B would consider

that people on floor C would have a positive gravitational potential energy because they are higher up the cliff.

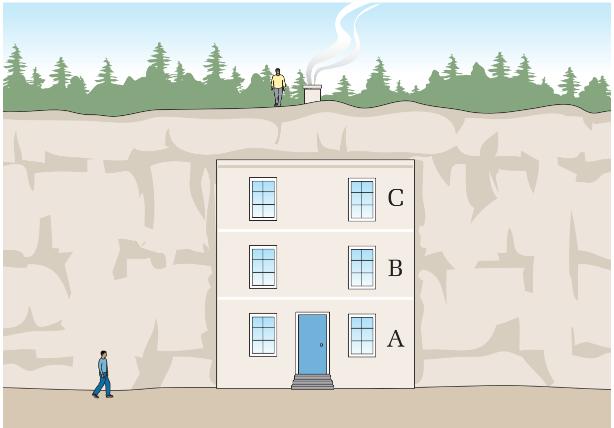



**Figure 6.3** Some houses in the Loire Valley are carved out of limestone cliffs.

Naturally, the person standing on the roof beside the chimney would consider that everyone in the house had negative gravitational potential energy. All of the residents would agree, however, on the amount of work that it took to carry a chair up from floor A to floor C, so the energy change would remain the same, regardless of the observer's level. At the same time, a book dropped from a window in floor B would hit the ground with the same kinetic energy, regardless of the location of the zero level for gravitational potential energy.

Figure 6.4 is a graph of the gravitational potential energy of a 1.0 kg object as it moves away from Earth's surface. Since work must be done on that object to increase the separation, the object is often referred to as being in a gravitational potential energy "well."

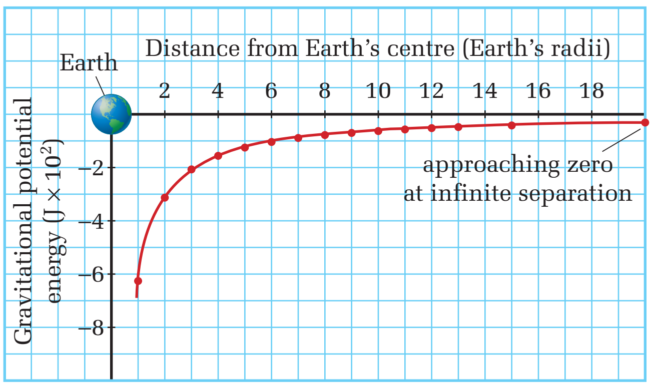



**Figure 6.4** Since work must be done on the 1.0 kg object to move it away from Earth, although the gravitational potential energy is always negative, it is increasing (becoming less negative) as it retreats farther and farther from Earth.

In summary, the energies of orbiting objects can be expressed as

$$E_g = -\frac{GMm}{r}$$

$$E_k = \frac{GMm}{2r}$$

Adding, you obtain

$$E_{total} = E_k + E_g$$

$$E_{total} = \frac{GMm}{2r} + \left(-\frac{GMm}{r}\right)$$

$$E_{total} = -\frac{GMm}{2r}$$

The last equation, the **total orbital energy,** involves only the mechanical energies — gravitational potential energy and kinetic energy. Other forms of energy, such as thermal energy, are not considered unless the satellite comes down in flames through the atmosphere.

You can obtain key information by determining whether the total orbital energy of an object is positive, zero, or negative. First, consider the conditions under which an object would have zero total orbital energy around a central object, such as a planet or star.

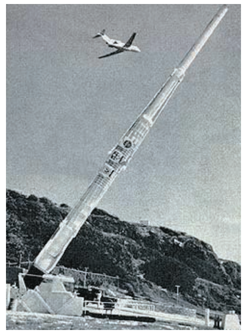
If an object is so far from Earth that gravity cannot pull it back, its gravitational potential energy is zero. If the object is motionless at that point, its kinetic energy is also zero, which gives a total energy of zero. The total orbital energy could also be zero if the magnitude of the kinetic and potential energies were equal. Under these conditions, the kinetic energy would be just great enough to carry the object to a distance at which gravity could no longer pull it back. It would then have no kinetic energy left and it would be motionless. By a similar analysis, you could draw all of the following conclusions.

- If the total of the kinetic and gravitational potential energies of an object is zero, it can just escape from the central object.
- If the total of the kinetic and gravitational potential energies of an object is greater than zero, it can escape from the central object and keep on going.
- If the total of the kinetic and gravitational potential energies of an object is less than zero, it cannot escape from the central object. It is said to be bound to the object.

The extra energy needed to free the object is called the binding energy. Since the object will be free with a total energy of zero, the binding energy is always the negative of the total energy: $E_{binding} = -E_{total}$.

## TECHNOLOGY LINK

Fire a cannon ball into space? In the early 1960s, Project HARP (High Altitude Research Project) did just that. Scientists at McGill University in Montréal welded two U.S. Navy cannon barrels together into a "super-gun" that fired 91 kg instrumentation packages to a height of more than 145 km from a launch site on the Caribbean island of Barbados.



HARP cannon

*Photo courtesy from the web sites: http://www.phy6.org/stargaze/Smartlet.htm and http://www-istp.gsfc.nasa.gov/stargaze/Smartlet.htm taken by Peter Millman.*

The orbital energy equations also have some informative simple relationships among themselves, as listed below.

- The magnitudes of the kinetic energy, the total energy, and the binding energy of an orbiting object are the same $$|E_k| = |E_t| = |E_{binding}|$$
- The magnitude of the gravitational potential energy is twice that of the other energies. $$|E_g| = 2|E_k| = 2|E_t| = 2|E_{binding}|$$
- If a satellite is in an orbit close to the planet, the radius of the orbit is essentially the same as the radius of the planet: $r_{orbit} \cong r_{planet} \cong r$. $$E_k = \frac{GMm}{2r}$$
- At the planet's surface, the energy needed to break free was seen in Section 6.1 to be $$E_{binding} = \frac{GMm}{2r}$$

By comparing the last two equations, you can see that the satellite in a circular orbit close to the planet already has half of the energy it needs to completely escape from that planet. The following problems will help you to develop a deeper understanding of orbital energies.

## SAMPLE PROBLEMS

### Space Problems

**1. On March 6, 2001, the Mir space station was deliberately crashed into Earth. At the time, its mass was $1.39 \times 10^3$ kg and its altitude was 220 km.**

**(a) Prior to the crash, what was its binding energy to Earth?**

**(b) How much energy was released in the crash? Assume that its orbit was circular.**

### Conceptualize the Problem

- When Mir was in Earth orbit, it had *kinetic* and *gravitational potential energy*, both of which are determined by its *orbital radius*.
- Mir's *binding energy* is the *negative* of its *total energy*.
- After the crash, Mir had *zero kinetic energy*.

- The law of *conservation of energy* applies; therefore, the energy released in the crash is the *difference* between the *total energy in orbit* and the *total energy* when resting on *Earth's surface*.

**Identify the Goals**

The binding energy, $E_{binding}$, of the Mir space station to Earth

The energy released during the crash of the Mir space station

## Identify the Variables and Constants

| Known | Implied | Unknown |
|---|---|---|
| $m = 1.39 \times 10^3$ kg (Mir) | $G = 6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$ | $E_{total}$ in orbit |
| $h = 2.20 \times 10^5$ m | $M = 5.978 \times 10^{24}$ kg (Earth) | $E_{binding}$ in orbit |
| | $r_{Earth} = 6.378 \times 10^6$ m | $E_g$ on ground |
| | | $\Delta E_{total}$ |

## Develop a Strategy

Determine the orbital radius.

$$r_{orbit} = r_{earth} + h_{from\ Earth}$$
$$r_{orbit} = 6.378 \times 10^6\ \text{m} + 2.20 \times 10^5\ \text{m}$$
$$r_{orbit} = 6.598 \times 10^6\ \text{m}$$

Calculate the total orbital energy before the crash.

$$E_t = -\frac{GMm}{2r}$$
$$E_t = -\frac{\left(6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(5.978 \times 10^{24}\ \text{kg}\right)\left(1.39 \times 10^3\ \text{kg}\right)}{2\left(6.598 \times 10^6\ \text{m}\right)}$$
$$E_t = -4.2019 \times 10^{10}\ \text{J}$$

Binding energy is the negative of the total energy.

$$E_{binding} = +4.2019 \times 10^{10}\ \text{J}$$
$$E_{binding} \cong +4.20 \times 10^{10}\ \text{J}$$

**(a)** The binding energy of the Mir space station in orbit was $4.20 \times 10^{10}$ J.

Calculate the mechanical energy after the crash. Note that when Mir was on Earth's surface, its kinetic energy was zero.

$$E_k = 0\ \text{J}$$
$$E_g = -\frac{GMm}{r}$$
$$E_g = -\frac{\left(6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(5.978 \times 10^{24}\ \text{kg}\right)\left(1.39 \times 10^3\ \text{kg}\right)}{6.378 \times 10^6\ \text{m}}$$
$$E_g = -8.6938 \times 10^{10}\ \text{J}$$
$$E_{total} = E_k + E_g$$
$$E_{total} = 0\ \text{J} - 8.6938 \times 10^{10}\ \text{J}$$
$$E_{total} = -8.6938 \times 10^{10}\ \text{J}$$

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Determine the difference in total energy before and after the crash.

$$\Delta E = E'_{\text{total}} - E_{\text{total}}$$
$$\Delta E = -8.6938 \times 10^{10}\ \text{J} - (-4.2019 \times 10^{10}\ \text{J})$$
$$\Delta E = -4.4919 \times 10^{10}\ \text{J}$$
$$\Delta E \cong -4.49 \times 10^{10}\ \text{J}$$

**(b)** When Mir crashed, $4.49 \times 10^{10}$ J of energy were released into the environment.

## Validate the Solution

The final answer is negative, which indicates a decrease in the energy of the system or a loss of energy to the environment.

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**2. A 4025 kg spacecraft (including the astronauts) is in a circular orbit 256 km above the lunar surface. Determine**

**(a) the kinetic energy of the spacecraft**

**(b) the total orbital energy of the spacecraft**

**(c) the binding energy of the spacecraft**

**(d) the speed required for escape**

## Conceptualize the Problem

- The spacecraft is in a circular orbit around the Moon, so the *Moon* is the *central* body.
- The spacecraft is *moving*, so it has *kinetic energy*.
- The spacecraft is in *orbit*, so it has *gravitational potential energy*.
- *Binding energy* is the amount of energy necessary to escape the *gravitational* pull of the *central body*.
- To escape a central body, a spacecraft must *increase* its *kinetic energy* until the *total energy* is *zero*.
- If you know the *kinetic energy*, you can find *speed*.

### Identify the Goals

**(a)** The kinetic energy of the spacecraft, $E_k$

**(b)** The total orbital energy of the spacecraft, $E_t$

**(c)** The binding energy of the spacecraft, $E_{\text{binding}}$

**(d)** The speed required for escape, $v_{\text{escape}}$

## Identify the Variables and Constants

| Known | Implied | Unknown |
|---|---|---|
| $m = 4025$ kg | $G = 6.673 \times 10^{-11}\ \dfrac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$ | $r_{\text{orbit}}$ |
| $h = 256$ km | $r_{\text{Moon}} = 1.738 \times 10^6$ m | $E_k$ |
| | $M_{\text{Moon}} = 7.36 \times 10^{22}$ kg | $E_t$ |
| | | $E_{\text{binding}}$ |
| | | $v_{\text{escape}}$ |

## Develop a Strategy

Determine the orbital radius of the spacecraft.

$$r_{\text{orbit}} = r_{\text{Moon}} + h$$
$$r_{\text{orbit}} = 1.738 \times 10^6\ \text{m} + 0.256 \times 10^6\ \text{m}$$
$$r_{\text{orbit}} = 1.994 \times 10^6\ \text{m}$$

Calculate the orbital kinetic energy.

$$E_{\text{k}} = \frac{GMm}{2r}$$
$$E_{\text{k}} = \frac{\left(6.673 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(7.36 \times 10^{22}\ \text{kg})(4025\ \text{kg})}{2(1.994 \times 10^6\ \text{m})}$$
$$E_{\text{k}} = 4.9569 \times 10^9\ \text{J}$$
$$E_{\text{k}} \cong 4.96 \times 10^9\ \text{J}$$

**(a)** The kinetic energy of the spacecraft is $4.96 \times 10^9$ J.

Total orbital energy is the negative of the kinetic energy.

$$E_{\text{t}} = -E_{\text{k}}$$
$$E_{\text{t}} = -4.96 \times 10^9\ \text{J}$$

**(b)** The total energy of the spacecraft is $-4.96 \times 10^9$ J.

The binding energy is the negative of the total energy

$$E_{\text{binding}} = -E_{\text{t}}$$
$$E_{\text{binding}} = -(-4.952 \times 10^9\ \text{J})$$
$$E_{\text{binding}} = 4.96 \times 10^9\ \text{J}$$

**(c)** The binding energy of the spacecraft is $4.96 \times 10^9$ J.

The binding energy must come through additional kinetic energy

$$E'_{\text{k}} = E_{\text{k}} + E_{\text{binding}}$$
$$E'_{\text{k}} = 4.9569 \times 10^9\ \text{J} + 4.9569 \times 10^9\ \text{J}$$
$$E'_{\text{k}} = 9.9138 \times 10^9\ \text{J}$$

Find the speed from the kinetic energy.

$$\frac{1}{2}mv^2 = E_{\text{k}}$$
$$v = \sqrt{\frac{2E_{\text{k}}}{m}}$$
$$v = \sqrt{\frac{2(9.9138 \times 10^9\ \text{J})}{4025\ \text{kg}}}$$
$$v = 2.2195 \times 10^3\ \frac{\text{m}}{\text{s}}$$
$$v \cong 2.22 \times 10^3\ \frac{\text{m}}{\text{s}}$$

**(d)** The escape speed for the spacecraft is $2.22 \times 10^3$ m/s.

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### Validate the Solution

The escape speed for the spacecraft could have been determined from the equation $v_{escape} = \sqrt{\frac{2GM}{r}}$. Carrying out this calculation yields the same escape speed. Since it agrees with the results of the energy calculations, it acts as a check on the first three answers.

## PRACTICE PROBLEMS

**4.** A 55 kg satellite is in a circular orbit around Earth with an orbital radius of $7.4 \times 10^6$ m. Determine the satellite's

**(a)** kinetic energy

**(b)** gravitational potential energy

**(c)** total energy

**(d)** binding energy

**5.** A 125 kg satellite in a circular orbit around Earth has a potential energy of $-6.64 \times 10^9$ J. Determine the satellite's

**(a)** kinetic energy

**(b)** orbital speed

**(c)** orbital radius

**6.** A 562 kg satellite is in a circular orbit around Mars. Data: $r_{Mars} = 3.375 \times 10^6$ m; $r_{orbit} = 4.000 \times 10^6$ m; $M_{Mars} = 6.420 \times 10^{23}$ kg

**(a)** If the satellite is allowed to crash on Mars, how much energy will be released to the Martian environment?

**(b)** List several of the forms that the released energy might take.

**7.** From the orbital kinetic energy of the lunar spacecraft in the second sample problem, determine its orbital speed. What increase beyond that speed was required for escape from the Moon?

**8.** A 60.0 kg space probe is in a circular orbit around Europa, a moon of Jupiter. If the orbital radius is $2.00 \times 10^6$ m and the mass of Europa is $4.87 \times 10^{22}$ kg, determine the

**(a)** kinetic energy of the probe and its orbital speed

**(b)** gravitational potential energy of the probe

**(c)** total orbital energy of the probe

**(d)** binding energy of the probe

**(e)** *additional* speed that the probe must gain in order to break free of Europa

**9.** A $1.00 \times 10^2$ kg space probe is in a circular orbit, 25 km above the surface of Titan, a moon of Saturn. If the radius of Titan is 2575 km and its mass is $1.346 \times 10^{23}$ kg, determine the

**(a)** orbital kinetic energy and speed of the space probe

**(b)** gravitational potential energy of the space probe

**(c)** total orbital energy of the space probe

**(d)** binding energy of the space probe

**(e)** *additional* speed required for the space probe to break free from Titan

**10.** Material has been observed in a circular orbit around a black hole some five thousand light-years away from Earth. Spectroscopic analysis of the material indicates that it is orbiting with a speed of $3.1 \times 10^7$ m/s. If the radius of the orbit is $9.8 \times 10^5$ m, determine the mass of the black hole.

## 6.2 Section Review

1. C Explain why the determination of orbital speed does not require knowledge of the satellite's mass, while determination of orbital energies does require knowledge of the satellite's mass.

2. K/U Explain why the binding energy of a satellite is the negative of its total orbital energy. Why does this relationship not depend on the satellite being in a circular orbit?

3. C Draw a concept organizer to show the links between the general equations for work and energy and the orbital energy equations. Indicate in the organizer which equations are joined together to produce the new equation.

4. I A satellite with an orbital speed of $v_{\mathrm{orbit}}$ is in a circular orbit around a planet. Prove that the speed for a satellite to escape from orbit and completely leave the planet is given by $v_{\mathrm{escape}} = \sqrt{2}(v_{\mathrm{orbit}})$.

5. MC The magnitude of the attractive force between an electron and a proton is given by $F = \frac{kq_{\mathrm{e}}q_{\mathrm{p}}}{r^2}$, where $q_{\mathrm{e}}$ is the magnitude of the charge on the electron, $q_{\mathrm{p}}$ is the magnitude of the charge on the proton, $r$ is the separation between them, and $k$ is a constant that plays the same role as $G$. If the mass of the electron is represented by $m_{\mathrm{e}}$, derive an equation for the orbital speed of the electron.